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Multi-Objective Optimization of Perturbed Impulsive Rendezvous Trajectories Using Physical Programming

Ya-zhong Luo* and Guo-jin Tang†
National University of Defense Technology, 410073 Changsha, People's Republic of China
and
Geoff Parks‡
University of Cambridge, Cambridge, England CB2 1PZ, United Kingdom

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I. Introduction

RECENTLY, it has been shown that taking the total velocity characteristic, the time of flight, and the trajectory safety into consideration and constructing a multi-objective optimization problem is an attractive and realistic proposition for rendezvous trajectory design [1,2]. Luo et al. [1] formulated the multi-objective linearized rendezvous optimization problem and solved it through the multi-objective genetic algorithm NSGA-II. It was shown that the tradeoffs between time of flight, propellant cost, and trajectory safety are quickly established using NSGA-II. In recognition of the drawbacks associated with linearized rendezvous equations, this study was expanded [2] to a nonlinear two-body rendezvous by using NSGA-II and a Lambert algorithm. The nonlinear two-body multi-objective model is more accurate and suitable for more problems in comparison with linearized rendezvous models, which are limited to circular and near-rendezvous. However, the two-body model still does not take into account trajectory perturbations, such as nonspherical perturbations and atmospheric drag, which exist in real operational missions. Thus, it is desirable to be able to obtain Pareto-optimal solutions for perturbed rendezvous trajectories.

Multi-objective design optimization always tries to locate a Pareto-optimal solution set. In previous multi-objective design optimization studies for spacecraft trajectories [3,4], it was clearly stated that feasible solution-generation software was a critical factor to the success of multi-objective evolutionary algorithms, but, as is well-known, obtaining a feasible solution for a spacecraft trajectory is often very difficult. In linearized and two-body multi-objective rendezvous problems [1,2], the introduction of the Clohessy–Wiltshire algorithm and a Lambert algorithm ensures that each solution naturally satisfies the rendezvous conditions. This contributes significantly to the success of NSGA-II.

At present, there is no easy and quick method for providing one feasible solution for a nonlinear perturbed rendezvous problem. The optimization approach for perturbed rendezvous proposed in [5] locates a feasible solution at a CPU cost of 5 s on a 2.8 GHz computer. If we introduce this approach into the multi-objective design optimization of perturbed rendezvous and NSGA-II is terminated after 20,000 function evaluations, then the total computation time will be about 28 h. For a trajectory design problem, this much computation time is not ideal.

Because the perturbed rendezvous model is very similar to the two-body rendezvous model, we can assume that the tradeoffs between the performance indices of a perturbed rendezvous trajectory are similar to those of a two-body rendezvous trajectory, which have been demonstrated by a two-body Pareto-optimal solution set, but the single Pareto-optimal solution chosen will evidently be somewhat different. For preliminary rendezvous mission design, the tradeoffs demonstrated by the two-body Pareto-optimal solution set are valuable, but for practical mission design, locating a designer-preferred solution that takes into account the practical trajectory perturbations is more important than finding the entire Pareto solution set. Thus, we judge the most important task for multi-objective optimization of perturbed rendezvous to be to develop an approach that can locate a designer-preferred solution that can be directly applied to a mission plan.

The main goal of this Note is to employ physical programming as the base method to develop an optimization approach for obtaining a tradeoff solution for multi-objective nonlinear perturbed rendezvous. Physical programming was initially developed by Messac [6] as an effective multi-objective and design optimization method, and it has been applied successfully in diverse areas of engineering and operations research [7–9]. However, it has been less applied in spacecraft trajectory design. The main contributions of this Note are twofold:

1) Extension of the multi-objective optimization of rendezvous from linearized and two-body models to a nonlinear perturbed model.

2) As far as we know, this is the first time that physical programming has been applied to spacecraft rendezvous trajectory design, and a quick global optimization approach combining physical programming and simulated annealing to the multi-objective optimization of perturbed rendezvous is examined.

II. Multi-Objective Optimization Model

A. Multi-Objective Optimization Function

The time of flight is the first objective function:

$$\min f_1(\mathbf{x}) = t_f - t_0 \tag{1}$$

The total velocity characteristic is the second objective function:

$$\min f_2(\mathbf{x}) = \Delta v = \sum_{i=1}^{n} |\Delta \mathbf{v}_i| \tag{2}$$

where $\Delta \mathbf{v}_i$ is the impulse vector.

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*Lecturer, College of Aerospace and Material Engineering; yzluo@sohu.com. Member AIAA (Corresponding Author).
†Professor, College of Aerospace and Material Engineering.
‡Senior Lecturer, Engineering Design Center, Department of Engineering.

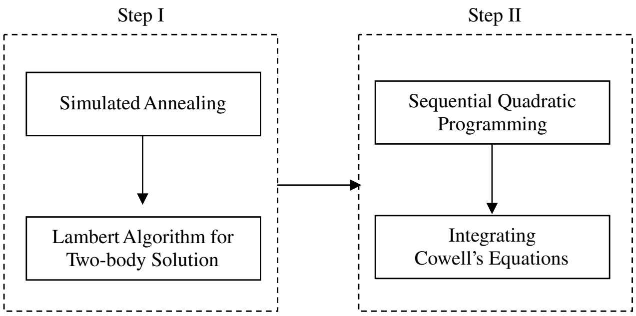

Fig. 1 Solution framework for optimizing perturbed rendezvous trajectories.

The trajectory safety-performance index is the third objective function:

$$\max f_3(\mathbf{x}) = \mathbf{r}_{\text{safe}} \tag{3}$$

The definition and calculation method of $\mathbf{r}_{\text{safe}}$ are provided in [1,2]. Because the time of rendezvous flight is chosen as an objective function, t_f is an optimization variable. The impulse times t_i $(i = 1, 2, \ldots, n)$ and impulse vectors $\Delta\mathbf{v}_i$ $(i = 1, 2, \ldots, n)$ are also optimization variables. There are small differences in these optimization variables for different models. For the two-body model, the optimization variable vector $\mathbf{x}$ is

$$\mathbf{x} = (t_f, t_1, t_2, \ldots, t_n, \Delta\mathbf{v}_1, \Delta\mathbf{v}_2, \ldots, \Delta\mathbf{v}_n) \tag{4}$$

And for the perturbed model,

$$\mathbf{x} = (t_f, t_1, t_2, \ldots, t_n, \Delta\mathbf{v}_1, \Delta\mathbf{v}_2, \ldots, \Delta\mathbf{v}_n) \tag{5}$$

where n is the number of impulses. More details on how to choose the optimization variables can be found in [10]. Constraints on impulse times are considered. As in [1,2], the interval between two arbitrary impulses should be larger than Δt, which is a user-defined time interval.

B. Optimization Approach

The physical programming method involves converting a multi-objective problem into a single-objective problem by using preference functions that capture the designer's preferences and then solving this single-objective optimization to find a compromise solution. The physical programming formulation considers the tradeoffs that exist between objectives implicitly and generates a Pareto design that is in the preferred region of the Pareto surface. Details of the physical programming approach can be found in [6–9].

First, we employ the physical programming approach to convert the multi-objective functions into a single-objective function:

$$\min P = \frac{1}{n_{\text{sc}}} \sum_{i=1}^{n_{\text{sc}}} p_i(f_i(\mathbf{x})) \tag{6}$$

where n_{sc} is the number of soft metrics associated with the problem, and p_i are preference functions.

After the single-objective optimization problem is formulated, a global efficient method for solving the perturbed approach problem is employed that is similar to that used in [10]. In this approach, a feasible iteration optimization model is first formulated using a Lambert algorithm and a simulated annealing algorithm is employed to locate the unperturbed two-body solution. Subsequently, an infeasible iteration optimization model accounting for trajectory perturbations is formulated, and a sequential quadratic programming algorithm is used to obtain the perturbed solution with the two-body solution as an initial reference solution. The hybrid approach for locating a feasible solution for a perturbed impulsive rendezvous is a two-step strategy, as illustrated by Fig. 1.

Although the optimal multi-objective multiple-impulse rendezvous problem is formulated as an unconstrained problem using the Lambert algorithm, classical nonlinear optimization algorithms such as sequential quadratic programming (SQP) and the simplex method (SM) algorithm exhibit very poor performance in solving this problem, as observed in our numerical experiments. To overcome these problems, the simulated annealing (SA) algorithm [11], a stochastic optimization algorithm with global convergence capability, is employed to solve this nonconvex optimization problem. The SA algorithm implementation is the same as that used in [12]. This implementation is designed for continuous optimization, and it has been shown to offer better performance than other popular optimization algorithms, including genetic algorithms, in solving both functional and spacecraft trajectory optimization problems.

III. Examples

Homing rendezvous missions are selected to demonstrate our approach. The major objective of the homing rendezvous phase is the reduction of trajectory dispersions (i.e., the achievement of position, velocity, and angular rate conditions necessary for the initiation of the close-range rendezvous operations). Passive trajectory safety is one important performance index when designing a homing rendezvous trajectory.

Here, a near-circular rendezvous mission is considered. The target orbit is elliptical, with an apogee height of 355 km and perigee height of 340 km. The initial conditions in the target orbit frame [1] are taken to be 35,713.3 m, $-13,455.1$ m, -28.8 m, -22.9 m/s, 0.66 m/s, and -0.09 m/s, and the final conditions are 2000 m, 0, 0, 0, 0, 0, with $\Delta t = 100$ s. The SA optimization is terminated after 20,000 function evaluations.

Table 1 provides details of the preference objective function value ranges used. Table 2 shows the optimization results obtained for different trajectory models.

Our numerical experiments show that obtaining a good solution for the proposed physical programming-based problem of the three-objective design optimization of perturbed impulsive rendezvous is very difficult. Several popular optimization algorithms, including a real-valued genetic algorithm (GA), SA, SM, and SQP, were tested on this problem. Table 3 lists the best two-body solutions obtained by the different algorithms in ten independent runs. Table 4 presents statistical performance measures for the different algorithms. From Tables 3 and 4, it is found that even SA and GA cannot be guaranteed to locate the best solution in all runs, to say nothing of the classical optimization algorithms.

In previously published papers on physical programming [6–9], no special attention was given to the optimization algorithms used and, generally, classical algorithms were employed. The present study shows that, at least for some applications, it is necessary to employ a global optimization algorithm in the physical programming-based optimization framework to reliably to locate a desired Pareto-optimal solution. Combining physical programming

Table 1 Preference structure for the design objectives

Objective	Class type	Highly desirable	Desirable f_{i1}	Tolerable f_{i2}	Undesirable f_{i3}	Highly undesirable f_{i4}	f_{i5}
Δv, m/s	1-S	0	15	22	32	42	50
t_f, s	1-S	0	1500	2000	2600	3000	4000
$\mathbf{r}_{\text{safe}}$, m	2-S	$+\infty$	2000	1500	1000	600	100

Table 2 Optimization results for different trajectory models.

		Objective function values		
Trajectory model	Aggregate preference function, Eq. (6)	Δv, m/s	t_f, s	$\mathbf{r}_{\mathrm{safe}}$, m
Two-body	0.203	16.9984	1878.63	1908.18
J_2	0.199	16.9211	1892.71	1978.87
$J_2 + J_3 + J_4$	0.198	16.9337	1892.71	1999.09
$J_2 + J_3 + J_4 +$ atmospheric drag	0.198	16.9338	1892.71	1998.92

Table 3 Best two-body solutions obtained by different optimization algorithms

		Objective function values		
Optimization algorithms	Aggregate preference function, Eq. (6)	Δv, m/s	t_f, s	$\mathbf{r}_{\mathrm{safe}}$, m
SA	0.203	16.9984	1878.63	1908.18
GA	0.212	17.2635	1894.75	1908.19
SM	0.207	17.9673	1842.07	1902.10
SQP	30.046	67.8113	3059.67	1325.27

Table 4 Statistical performance measures of different algorithms for two-body solutions

	Aggregate preference function, Eq. (6)			
Algorithms	Best	Worst	Mean	Standard deviation
SA	0.203	0.216	0.209	1.168e − 5
GA	0.212	0.270	0.226	2.242e − 4
SM	0.207	60.959	15.193	5.254e + 2
SQP	30.046	235.030	118.112	4.470e + 3

with popular evolutionary algorithms is desirable and necessary for practical applications.

To verify the Pareto optimality of the physical programming solution, it is compared with the Pareto-optimal solutions obtained using the NSGA-II approach [2] in Fig. 2. Figure 2 presents the two-body solution. Evidently, the single solution obtained using the physical programming approach is closer to the true Pareto frontier than the set of solutions obtained using NSGA-II. Even a time-fixed propellant-optimal multi-impulse rendezvous trajectory design problem is very difficult, to say nothing of the multi-objective design optimization of multi-impulse rendezvous trajectories. Because the NSGA-II approach locates hundreds of solutions in one run, and is indeed designed to do so, it inevitably places a greater emphasis on exploration (of the design space) at the expense of exploitation (the improvement of promising solutions) during the search. Conversely, because the physical programming approach is designed to locate a single optimal solution in one run, it offers better exploitation capability.

The physical programming approach is effective in providing a desired Pareto-optimal solution in the design space of the perturbed rendezvous trajectory problem. The objective function values are all in the desirable range. After optimization, if the designers wish to explore other possibilities, they simply need to change the region limits defined in the preference table. The fact that the region limits are physically meaningful to designers provides a flexibility that is not available in classical weighted-sum aggregation approaches. As discussed in detail in previous physical programming papers, changing the numbers in the preference table to explore the design space is fundamentally more deterministic than iterating on numerical weights in the aggregate objective function. For example, if we relax the performance index of the time of flight and change its preference ranges to 0, 2500, 3000, 3600, 4000, or 5000 s,

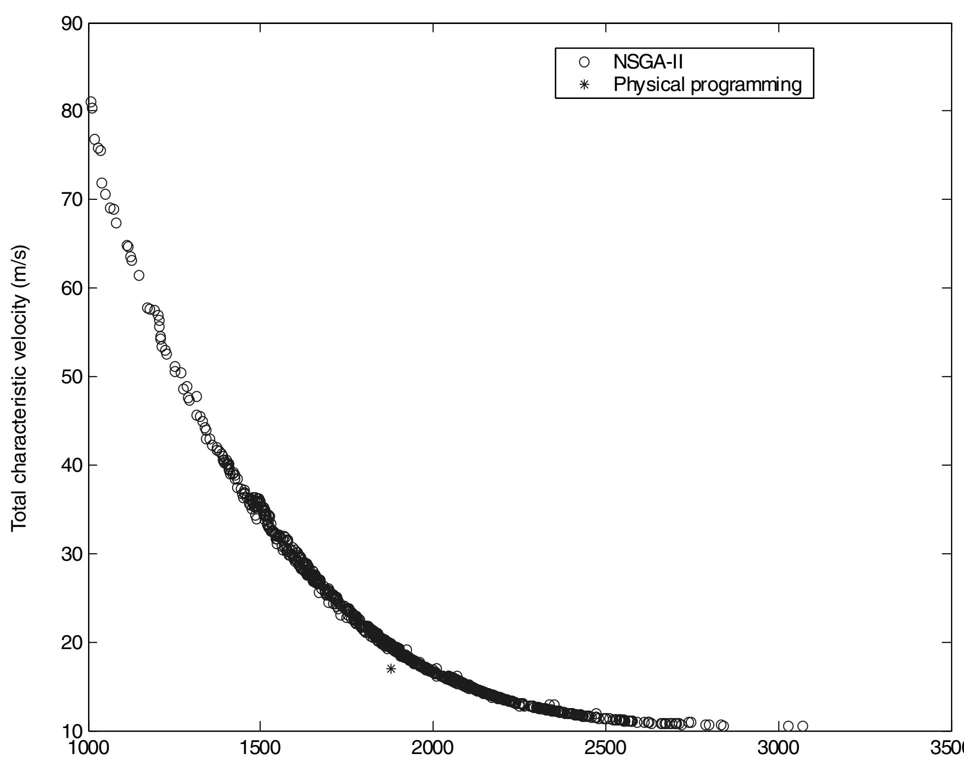

Fig. 2 Pareto-optimal solutions obtained by NSGA-II and physical programming.

we can then obtain a new designer-preferred solution of $\Delta v = 13.5174$ m/s, $t_f = 2135.75$ s, and $\mathbf{r}_{\text{safe}} = 1939.5$ m. Both Δv and t_f are in the highly desirable range, and $\mathbf{r}_{\text{safe}}$ is in the desirable range.

IV. Conclusions

The present optimal impulsive rendezvous design approach incorporates three objective functions, including minimum propellant cost, minimum time of flight, and maximum trajectory safety with consideration of trajectory perturbations. An optimization approach using physical programming, a Lambert rendezvous algorithm, and simulated annealing for multi-objective optimization of nonlinear perturbed rendezvous is proposed. It is found that the designer-preferred solution, which can be directly applied to a mission plan, is reliably identified. In this Note, we successfully demonstrate the application of the physical programming method for the generation of efficient desired design solutions of a perturbed rendezvous trajectory problem through the example of a homing rendezvous problem. We demonstrate that in addition to the flexibility it provides in problem formulation, the physical programming method also appears to be superior to evolutionary multi-objective optimization methods in locating Pareto-optimal solutions closer to the true Pareto frontier. Also, we observe that to solve such physical programming-based optimization problems reliably, it is necessary to introduce global heuristic optimization algorithms, such as genetic algorithms or simulated annealing. It is beneficial to apply the physical programming method together with heuristic optimization algorithms to complex spacecraft trajectory design problems with multiple performance attributes for which obtaining a feasible solution is always difficult and computationally costly.

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